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Global-Local Observer Markov Parameter Identification for System Realization

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The observer/Kalman filter identification method uses general input/output data to compute the Markov parameters of an asymptotically stable observer. A state space realization is subsequently obtained from these estimated Markov parameters. However, numerical ill-conditioning, data acquisition, and computer memory constraints may limit the number of measurements that can be processed for Markov parameter estimation. Use of limited measurements then will reduce spatial information. A global-local version of the observer/Kalman filter identification was therefore developed to overcome this dilemma. This method obtains a local state space realization from a reduced set of measurements. A global state space realization is subsequently obtained from a least squares process on the remaining measurements. A six-degree-of-freedom numerical simulation and experimental data from a frame structure are presented to validate the proposed method.

Introduction

MANY frequency and time domain methods have been formulated for system realization.[1] In particular, a time domain method called observer/Kalman filter identification (OKID)[2] will be considered. This method uses general input/output data to obtain the Markov parameters of an asymptotically stable observer. The system Markov parameters are then determined recursively from the Markov parameters of the observer system, from which a state space realization is obtained. However, this method can lead to a numerically ill-conditioned matrix inverse if the measurements are not independent.[2] Even if the measurements are independent, it is known that an optimal number of outputs have the highest modal recovery.[3] In addition, the data acquisition system may limit the number of measurements during a test.

If limited channels exist in performing a modal test, one can argue for using frequency response measurements and subsequently advocating the frequency domain methods. Although this approach is valid, there are several problems to its use. First, multiple experiments need to be performed to average out the effects of noise in the frequency response functions (FRF). Second, and probably most importantly, the averaging process can take a long time, especially if there are many measurement locations. Furthermore, the use of a Fourier transformation may not be strictly valid under certain conditions and entails special postprocessing of the data (i.e., windowing) to prevent leakage, among other problems.[4] Finally, frequency domain methods inherently assume a linear structure because FRF are typically undefined for nonlinear systems.

Most forced response time domain methods, including OKID, require output measurements from a set of inputs. That is, all the outputs must be measured simultaneously. This is seldom possible when there is a limit on the number of available channels in the data acquisition system and supporting instrumentation. For example, a forced response modal test of a five-output one-input system entails recording five acceleration signals and the single input, which would require six channels. If only four channels are available, for example, such a test would not be possible in the OKID approach.

The global-local OKID (GLOKID) method is therefore proposed to circumvent this dilemma. The advantage is that the measurements can be acquired separately from different inputs. By computing a realization from a subset of outputs, system frequencies and damping can be obtained. Under the assumption that these frequencies represent the system globally, spatial information at the other outputs can be obtained. The idea behind GLOKID is that a global state space realization is readily available once a local state space realization is known. In this manner, only a subset of the most independent measurements will be used to obtain the local state-space model, which contains the assumed global modal information. The unprocessed measurements are then incorporated into the realization using a least squares procedure.

The idea of a reduced set of outputs for modal parameter identification is not new. Pappa et al.[5] discuss such an approach using the eigensystem realization algorithm (ERA). An interest in particular modes, a reduction in the size of data matrices, and numerical conditioning concerns are a few reasons for using a reduced measurement set.

Problem Formulation

The state-variable description for a discrete time, linear, finite-dimensional, time invariant dynamic system can be written as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \qquad k \geq 0 \tag{1}$$

where $A \in \Re(n, n)$ and $\Re(n, n)$ is the set of real $n \times n$ matrices, $B \in \Re(n, \mathrm{Ni})$, $C \in \Re(\mathrm{No}, n)$, and $D \in \Re(\mathrm{No}, \mathrm{Ni})$. The values No and Ni represent the number of outputs and inputs, respectively, and n is the system order (i.e., twice the number of vibration modes for an underdamped system). The system matrix A contains the modal information (frequency and damping), input matrix B determines the input location, output matrix C produces the physical measurements, and the direct transmission matrix D relates the input to the output.

An observer model for Eq. (1) is constructed by adding and subtracting the state term $Gy(k)$. Introducing the notation

$$\begin{aligned} \bar{A} &= A + GC, & \bar{B} &= [B + GD \quad -G] \\ \bar{D} &= [D \quad 0], & v(k) &= \begin{bmatrix} u(k) \\ y(k) \end{bmatrix} \end{aligned} \tag{2}$$

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yields the linear observer model

$$\begin{aligned} x(k+1) &= \bar{A}x(k) + \bar{B}\nu(k) \\ y(k) &= Cx(k) + \bar{D}\nu(k) \end{aligned} \qquad k \geq 0 \tag{3}$$

The matrix representation of the input/output time histories of Eqs. (3), assuming zero initial conditions and that $\bar{A}$ is asymptotically stable (i.e., $\bar{A}^k \approx 0$, $k \geq p$, where p is some sufficiently large integer), is

$$y_{\mathrm{No} \times \ell} = \bar{Y}_{\mathrm{No} \times [(\mathrm{Ni}+\mathrm{No})p+\mathrm{Ni}]} V_{[(\mathrm{Ni}+\mathrm{No})p+\mathrm{Ni}] \times \ell} \tag{4}$$

where

$$y = [y(0) \quad y(1) \quad y(2) \quad \cdots \quad y(\ell-1)]$$

$$\bar{Y} = [D \quad C\bar{B} \quad C\bar{A}\bar{B} \quad \cdots \quad C\bar{A}^{p-1}\bar{B}]$$

$$V = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(p) & \cdots & u(\ell-1) \\ 0 & \nu(0) & \nu(1) & \cdots & \nu(p-1) & \cdots & \nu(\ell-2) \\ 0 & 0 & \nu(0) & \cdots & \nu(p-2) & \cdots & \nu(\ell-3) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \nu(0) & \cdots & \nu(\ell-p-1) \end{bmatrix}$$

and ℓ is the data length with each zero in V representing a zero (Ni + No) column vector. The term $\bar{Y}$ is defined as the observer Markov parameters and can be solved for by a pseudoinverse of matrix V in Eq. (4). The next objective is to compute the system Markov parameters Y from the observer Markov parameters $\bar{Y}$. Because $Y_k = CA^{k-1}B$, let $\bar{Y}_k = C\bar{A}^{k-1}\bar{B}$ and define the following:

$$\begin{aligned} Y &\equiv [Y_0 \quad Y_1 \quad Y_2 \quad \cdots \quad Y_{\ell-1}] \\ &= [D \quad CB \quad CAB \quad \cdots \quad CA^{\ell-2}B] \\ \bar{Y} &\equiv [\bar{Y}_0 \quad \bar{Y}_1 \quad \bar{Y}_2 \quad \cdots \quad \bar{Y}_{\ell-1}] \\ &= [D \quad C\bar{B} \quad C\bar{A}\bar{B} \quad \cdots \quad C\bar{A}^{\ell-2}\bar{B}] \\ \bar{Y}_k &= \left[\bar{Y}_k^{(1)} \quad \bar{Y}_k^{(2)}\right], \qquad k = 1, \ldots, \infty \end{aligned} \tag{5}$$

The relationship between the observer and system Markov parameters can then be shown to be[2]

$$\begin{aligned} Y_k &= \bar{Y}_k^{(1)} + \sum_{i=1}^{k} \bar{Y}_k^{(2)} Y_{k-i}, \qquad k = 1, \ldots, p \\ Y_k &= \sum_{i=1}^{p} \bar{Y}_i^{(2)} Y_{k-i}, \qquad k = p+1, \ldots, \infty \end{aligned} \tag{6}$$

Therefore, solving Eq. (4) for $\bar{Y}$ using a pseudoinverse of matrix V, a state-space model (A, B, C, D) may then be realized from the sequence Y_k [obtained from Eqs. (6)] using either ERA[6] or ERA/data correlations.[7] However, it is important that the outputs be as linearly independent as possible to minimize any numerical ill-conditioning of the pseudoinverse of the V matrix in Eq. (4) (Ref. 2). That is, it is vital that the observer Markov parameters are accurately computed because they are used to obtain the system Markov parameters.

The global-local concept is now discussed. As stated previously, GLOKID begins with the premise that only a few outputs should be used for determination of $\bar{Y}$. Letting No* represent this reduced output set, Eq. (4) becomes

$$y_{\mathrm{No}^* \times \ell} = \bar{Y}_{\mathrm{No}^* \times [(\mathrm{Ni}+\mathrm{No}^*)p+\mathrm{Ni}]} V_{[(\mathrm{Ni}+\mathrm{No}^*)p+\mathrm{Ni}] \times \ell} \tag{7}$$

It is assumed that only the most linearly independent measurement data are included in No*. This will permit a well-conditioned solution for the observer Markov parameters in $\bar{Y}$. The procedure then is to solve for $\bar{Y}$ from Eq. (7), recover Y, and use ERA to realize a local state-space model of the system (A, B, C^*, D^*). It is a local representation because C^* and D^* are valid only for the No* outputs. It is assumed that A and B contain all of the information of interest (i.e., all the frequencies that would have been obtained from all No outputs can be obtained from the No* outputs). Because only No* outputs are processed for Markov parameter estimation, $\overline{\mathrm{No}}$ (i.e., $\overline{\mathrm{No}} = \mathrm{No} - \mathrm{No}^*$) outputs have yet to be used. It is then necessary to extend the local state space representation to a global realization, which includes the $\overline{\mathrm{No}}$ outputs. The following derivation provides this extension.

The matrix representation of the input/output time histories of Eqs. (1) for the unprocessed $\overline{\mathrm{No}}$ measurements, assuming zero initial conditions, can be written as

$$y_{\overline{\mathrm{No}} \times \ell} = Y_{\overline{\mathrm{No}} \times (\mathrm{Ni})\ell} U_{(\mathrm{Ni})\ell \times \ell} \tag{8}$$

Using matrix manipulations on the preceding equation,

$$y = [\bar{D} \quad \bar{C}B \quad \bar{C}AB \quad \cdots \quad \bar{C}A^{\ell-2}B] \times \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(\ell-1) \\ 0 & u(0) & u(1) & \cdots & u(\ell-2) \\ 0 & 0 & u(0) & \cdots & u(\ell-3) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & u(0) \end{bmatrix} \tag{9}$$

$$y = \bar{D}u + \bar{C}[B \quad AB \quad \cdots \quad A^{\ell-2}B] \times \begin{bmatrix} 0 & u(0) & u(1) & u(2) & \cdots & u(\ell-2) \\ 0 & 0 & u(0) & u(1) & \cdots & u(\ell-3) \\ 0 & 0 & 0 & u(0) & \cdots & u(\ell-4) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & u(0) \end{bmatrix} \tag{10}$$

or

$$y = [\bar{D} \quad \bar{C}]\begin{bmatrix} u \\ U^* \end{bmatrix} = [\bar{D} \quad \bar{C}]\bar{U} \tag{11}$$

where

$$\bar{U} = \begin{bmatrix} u \\ U^* \end{bmatrix}, \qquad u = [u(0) \quad u(1) \quad \cdots \quad u(\ell-1)]$$

$$U^* = [B \quad AB \quad \cdots \quad A^{\ell-2}B] \times \begin{bmatrix} 0 & u(0) & u(1) & u(2) & \cdots & u(\ell-2) \\ 0 & 0 & u(0) & u(1) & \cdots & u(\ell-3) \\ 0 & 0 & 0 & u(0) & \cdots & u(\ell-4) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & u(0) \end{bmatrix}$$

and each zero in U^* represents a zero Ni column vector. Equation (11) can be solved for $\bar{C}$ and $\bar{D}$ from

$$[\bar{D} \quad \bar{C}] = y\bar{U}^+ = y\bar{U}^T(\bar{U}\bar{U}^T)^{-1} \tag{12}$$

where superscript + denotes a pseudoinverse and superscript T is a matrix transpose.

Conceptually, the use of Eq. (12) is straightforward, but it must be clarified. The A and B matrices, appearing in U^*, must be truncated to eliminate computational modes before evaluation of the $\bar{C}$ and $\bar{D}$ matrices. This truncation can be done by transforming the identified local system matrices to modal space:

$$\Lambda = \varphi^{-1}A\varphi, \qquad B_m = \varphi^{-1}B, \qquad C_m^* = C^*\varphi \tag{13}$$

where φ are the eigenvectors of A. In this form, computational modes can be eliminated judiciously by first deleting a row and column of Λ and then deleting the corresponding row in B_m and the corresponding column in C_m^*. Maintaining the same notation after model reduction, we rewrite Eq. (11) as

$$y = [\bar{D} \quad \bar{C}_m]\begin{bmatrix} u \\ U^* \end{bmatrix} = [\bar{D} \quad \bar{C}_m]\bar{U} \tag{14}$$

where the U^* matrix becomes

$$U^* = \begin{bmatrix} B_m & \Lambda B_m & \cdots & \Lambda^{\ell-2}B_m \end{bmatrix}$$
$$\times \begin{bmatrix} 0 & u(0) & u(1) & u(2) & \cdots & u(\ell-2) \\ 0 & 0 & u(0) & u(1) & \cdots & u(\ell-3) \\ 0 & 0 & 0 & u(0) & \cdots & u(\ell-4) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & u(0) \end{bmatrix} \tag{15}$$

and $\bar{C}_m$ is the modal space transformed $\bar{C}$ matrix. From a mathematical viewpoint, Eqs. (11) and (14) are equivalent because the modal space transformation is a similarity transformation (eigenvalues of A and Λ are the same) and $\bar{C}A^kB = \bar{C}_m\Lambda^kB_m$, $k \geq 0$. If it is necessary to eliminate modes from the A matrix, solving Eq. (14) results in estimating the mode shape matrix for the remaining outputs (i.e., $\bar{C}_m$). The state space realization for the entire system (all of the measurements) is then written as (Λ, B_m, C_m, D), where

$$C_m = \begin{bmatrix} C_m^* \\ \bar{C}_m \end{bmatrix}, \qquad D = \begin{bmatrix} D^* \\ \bar{D} \end{bmatrix} \tag{16}$$

If mode truncation is not needed, then solving Eq. (11) results in identifying the output state matrix for the remaining measurements. The realization in this case becomes (A, B, C, D), where

$$C = \begin{bmatrix} C^* \\ \bar{C} \end{bmatrix} \tag{17}$$

and the D matrix is the same as in Eq. (16). Note also that it is not necessary to perform the matrix multiplication indicated in Eq. (15). The columns can be recursively calculated from

$$U^*(k+1) = AU^*(k) + Bu(k), \qquad k \geq 0 \tag{18}$$

where $U^*(0) = 0_{n\times 1}$.

In summary, the computational steps are as follows: 1) select sensor subset No* and perform OKID-ERA or OKID-ERA/data correlations; 2) transform (A, B, and C^*) to modal form (Λ, B_m, C_m^*) and eliminate computational modes, if necessary; 3) calculate the appropriate $\bar{U}$ matrix with Eq. (15); and 4) solve Eq. (14) and get Eqs. (16). Equations (11) and (17) could also be used for a system without computational modes.

Results

The global-local method is first presented on a numerical simulation of a six-degree-of-freedom (DOF) lumped-mass model as shown in Fig. 1. The numerical values of the parameters are listed in Table 1. The viscous damping elements were of the Rayleigh type such that $C = \alpha M + \beta K$, where C, M, and K are the damping, mass, and stiffness matrices, respectively, and $\alpha = 0.2$ and $\beta = 0.0001$.

The simulation study generated 3000 data points at a sampling frequency of 20 Hz. The input, which was applied to the last mass ($m6$), was a zero mean, unit variance Gaussian random sequence. The first three acceleration outputs ($m1$, $m2$, $m3$) were used in GLOKID to determine system frequencies and damping. As stated by Juang et al.,[2] the observer decay value p needs to be four to five times the system order, so p was set to 60. The number of data points used to obtain the observer Markov parameters and in the

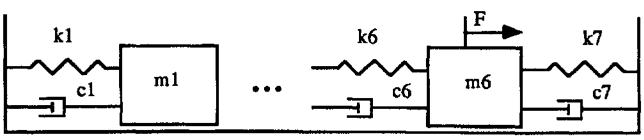


Fig. 1 Six-DOF lumped-mass model.

Table 1 Physical parameters for six-DOF system

Mass, kg	Stiffness, N/m
$m1 = 1.5$	$k1 = 1000$
$m2 = 0.1$	$k2 = 500$
$m3 = 0.4$	$k3 = 750$
$m4 = 2$	$k4 = 300$
$m5 = 0.8$	$k5 = 800$
$m6 = 1.3$	$k6 = 200$
——	$k7 = 400$

Table 2 Identified model parameters from six-DOF system using GLOKID

		Frequency, Hz		Damping		
Noise, %	Mode	Estimated	Exact	Estimated	Exact	MAC
1	1	1.4443	1.4442	0.0114	0.0115	1.0000
	2	3.3918	3.3920	0.0058	0.0058	0.9997
	3	4.1446	4.1446	0.0051	0.0051	0.9999
	4	6.1800	6.1800	0.0045	0.0045	1.0000
	5	6.7227	6.7227	0.0045	0.0045	1.0000
	6	18.7514	18.7505	0.0070	0.0067	0.9853
10	1	1.4424	1.4442	0.0171	0.0115	0.9998
	2	3.3930	3.3920	0.0078	0.0058	0.9978
	3	4.1444	4.1446	0.0050	0.0051	0.9992
	4	6.1796	6.1800	0.0048	0.0045	0.9999
	5	6.7232	6.7227	0.0045	0.0045	1.0000
	6	18.7687	18.7505	0.0078	0.0067	0.8655

least squares solution for the mode shapes at the last three outputs was 3000. And finally, the Hankel matrix size was 180×900. Typically, the number of rows of the Hankel matrix needs to be three to four times the system order, and the number of columns needs to be four to five times the number of rows, although these sizes can be reduced slightly when working with OKID.[3] Ultimately, however, computer memory limitations will be the deciding factor.

Noise levels of 1 and 10% were added to the output measurements to produce noisy data. Consequently, the signal-to-noise ratios [ratio of the root mean square (rms) of the noise-free measurements and rms of the noise] were around 50 and 5, respectively. The noise was zero mean and varying in amplitude from -1 to 1 times the maximum measurement amplitude. Table 2 presents the modal parameter comparisons. Observe that all the frequencies were identified solely from using the first three acceleration signals. This is a requirement for GLOKID. That is, the important frequencies need to be represented in the subset of measurements used to identify the local state space realization. In a practical application the recovered frequencies will significantly depend on which measurements are used. The global state-space model was subsequently obtained from Eq. (14).

The modal assurance criterion (MAC) in Table 2 is an orthogonality measure between the kth exact (ϕ_{ex}^k) and an identified mode shape (ϕ_{id}^k) and is calculated from

$$\text{MAC}_k = \frac{\left(\phi_{\text{ex}}^k\right)^T \phi_{\text{id}}^k}{\left|\phi_{\text{ex}}^k\right|\left|\phi_{\text{id}}^k\right|}, \qquad k = 1, \ldots, n \tag{19}$$

where $|\,|$ represents magnitude and superscript T denotes a matrix transpose. Table 2 shows that the modal parameters (i.e., frequency, damping, and mode shape) were well recovered for both noise levels. The sixth MAC value for the 10% noise case, however, shows a low correlation. This is due to the bias in the least squares solution in Eq. (14) when large measurement noise is present.[8]

GLOKID is next applied on an experimental three-dimensional frame structure. The test article, shown in Fig. 2 and schematically in Fig. 3, is a cantilevered frame structure of 2.5-m length (from base to tip). The bar elements and nodes are made of aluminum and manufactured by Mero Structures, Inc. The experiment consisted of one input and was applied at node 11 (z direction) with a stinger that was attached to an electromagnetic shaker. The input consisted of random noise generated by a Tektronix 2642A Fourier analyzer and was measured by a force transducer placed between the

Table 3 Identified frequency-damping values from frame structure

	GLOKID			ERA			OKID-FQ		
Mode	Freq., Hz	Damp	γ	Freq., Hz	Damp	γ	Freq., Hz	Damp	γ
1	22.35	0.0027	1.00	23.29	0.1783	0.43	22.65	0.0747	1.00
2	112.05	0.0036	1.00	111.88	0.0040	1.00	112.51	0.0027	1.00
3	193.01	0.0039	0.99	192.49	0.0020	0.99	193.28	0.0024	0.99

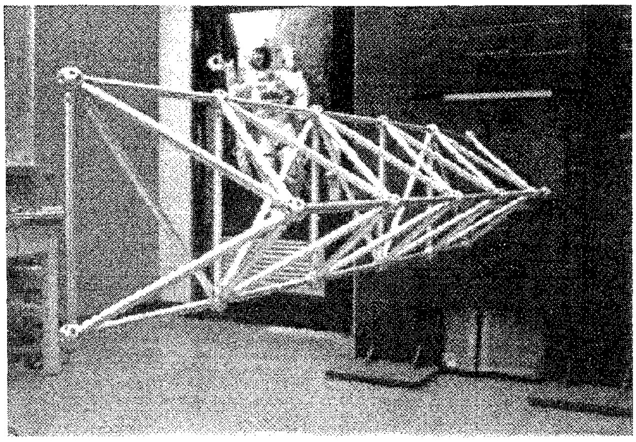


Fig. 2 Experimental test structure.

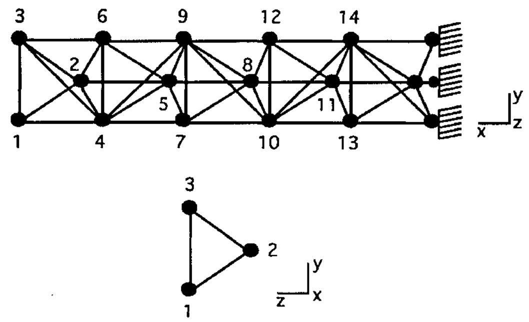


Fig. 3 Test structure schematic.

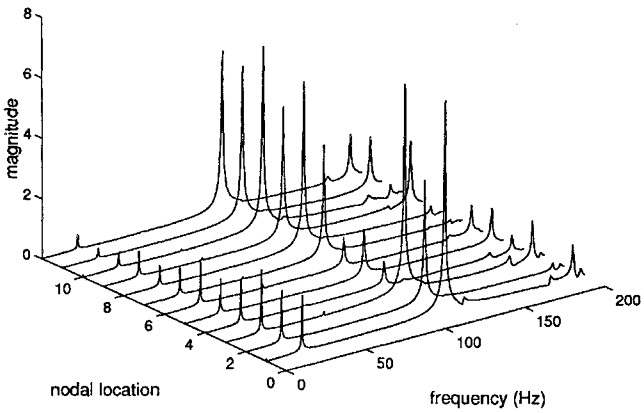


Fig. 4 Frame structure FRF.

node and stinger. For a proper OKID analysis, a 12-output single-input test is needed. This would require 13 channels when only two channels were available. Subsequently, 12 single-input/output forced response tests were performed. The input location was fixed throughout all the tests while the accelerometer was simply moved from point to point. The data were then processed with GLOKID. The sampling frequency was 512 Hz, and 4096 data points were collected from zero initial conditions.

The frame structure FRF are depicted in Fig. 4. It is expected that only the frame bending modes about the y axis will be excited because the excitation was applied in the z direction. These modes are clearly shown in Fig. 4 (i.e., the three largest peaks). However, some torsional modes were excited because the structure was not strictly symmetric. It is assumed, though, that the bending modes are sufficient to represent the motion at any output location.

Table 4 Variation of ERA frequencies and damping with system order

	System order = 20			System order = 30		
Mode	Freq., Hz	Damp	γ	Freq., Hz	Damp	γ
1	22.25	0.0084	0.99	22.31	0.0061	1.00
2	111.82	0.0034	1.00	111.94	0.0030	1.00
3	192.10	0.0035	0.99	192.80	0.0040	1.00

Table 5 MAC correlations between GLOKID mode shapes to ERA and OKID-FQ

Mode	GLOKID and ERA	GLOKID and OKID-FQ
1	1.00	0.95
2	0.98	0.92
3	0.89	0.78

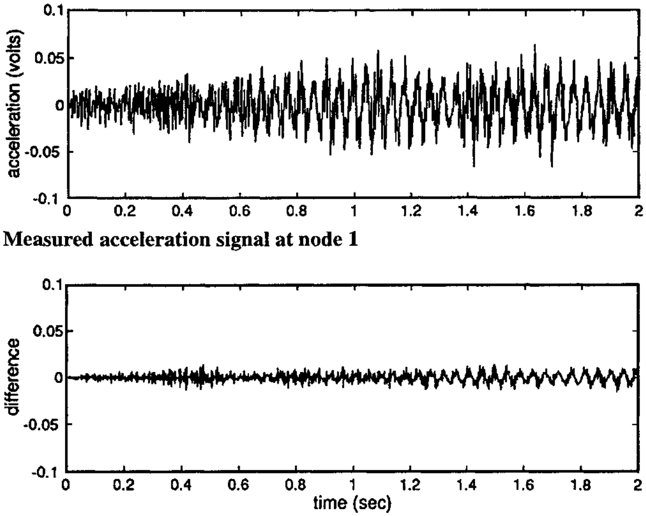
The parameter values in GLOKID were a p of 60 with 3000 data points. The Hankel matrix size was selected at 60×300. The single-input/output forced measurement at node 8 was used to obtain the system natural frequencies and damping (i.e., to obtain the local state-space model). Mode shape information was then extrapolated at the remaining 11 outputs (i.e., nodes 1–7, 9–12) using Eq. (14) with a data length of 3000 in the least squares process. A system order of 12 was selected. Two methods were used to verify the estimated global mode shapes. The first method inverted the 12 FRF and obtained pulse responses that were subsequently used in ERA. The Hankel matrix size was chosen at 60×300, and a system order of 12 was again selected. The second method was OKID-FQ,[1] which is a frequency domain version of OKID. It also processed the 12 FRF and obtained a state-space model. The parameter values for this case were a p of 10 with 1601 frequency data points and a Hankel matrix size of 120×361, where a system order of 12 was similarly selected.

Table 3 compares the frequency and damping estimates from all the methods. Notice that all gave excellent agreement in frequency and good estimates in damping. The only poor values were the damping for mode 1 from ERA and, to some extent, OKID-FQ. The reason for the poor ERA damping estimate was the low system order. A larger value would have allowed the noise to go to the computational modes instead of the system modes. This is shown in Table 4. The symbol γ denotes the modal amplitude coherence.[6] The value ranges from 0 to 1, where unity implies a perfect comparison and 0 implies no coherence. It is a frequency-damping estimator used to discriminate between computational and system modes.

The mode shape correlations between GLOKID and ERA and GLOKID and OKID-FQ are shown in Table 5. The correlation is the MAC value as defined earlier. Observe that there is excellent mode shape agreement between GLOKID and ERA for the first two modes, the third mode being only an approximation (albeit a good one). A similar result is seen between GLOKID and OKID-FQ with the exception of a poorer correlation in the third mode. To determine whether this is a result of the least squares process in GLOKID, the mode shape for mode 3 was compared between OKID-FQ and ERA. The correlation was 0.68 and indicates that this mode shape was poorly identified in OKID-FQ (assuming that

Table 6 Coherence function between experimental and reconstructed measurement data

Output location	1	2	3	4	5	6	7	8	9	10	11	12
cf	0.90	0.93	0.90	0.90	0.94	0.90	0.92	0.97	0.98	0.91	0.87	0.91



Measured acceleration signal at node 1

Difference between measured and reconstructed acceleration

Fig. 5 Experimental response (top) and reconstruction error (bottom) for node 1.

ERA gives the correct modal parameters). This lack of correlation might have been due to the low p value in OKID-FQ.

A final demonstration of the accuracy of the global state-space model is seen through a reconstruction of the experimental data. A time history comparison for the output at node 1 is shown in Fig. 5. The remaining outputs have similar plots, and quantification of this is presented in Table 6, which shows the coherence function for all 12 outputs. The coherence function cf is defined as the ratio of the mean square of the reconstructed data $\bar{y}$ to the mean square of the experimental data y, i.e.,

$$\text{cf} = \frac{\sum_{i=1}^{N} \bar{y}(i)^2}{\sum_{i=1}^{N} y(i)^2}$$

where N is the data length. If cf > 1, then let cf $= 1/\text{cf}$ so that it ranges from 0 to 1. A cf value of 1 means that there is a perfect fit to the experimental data, and a value of 0 implies that there is no relationship between the experimental and reconstructed data. The reason for the nonzero reconstruction error is the presence of the torsional modes in the data, which was not incorporated in the model. Despite this, the cf values are still good, as seen from Table 6.

Conclusions

A new version of the OKID method was presented. This method uses a limited number of measurements to identify a local state-space model. A global state space realization is then obtained by performing a least squares process on the remaining measurement set. The method was applied to simulated data from a six-DOF system and an experimental frame structure. The global-local approach has been found to be an effective means for modal parameter estimation when there are data acquisition limitations.

Acknowledgment

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